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CONTOUR DENSITIES USING DISCRETE PROBABILITY DISTRIBUTIONS

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Abstract: When the data collected or the experiment conducted is under ideal conditions, well-known statistical model may give a better fit. But when the situations are not ideal due to presence of some distorting factors, standard model may not give a good fit for the data set. One possible method to take an account of this distorting factor is to modify the existing model. In this article we generate densities based on the size of Contours of a discrete density function. As an Illustration contour densities of Geometric distribution have been discussed.

Keywords: Contour Transformation, Discrete distributions, Unimodal

Introduction:
Sometimes a proposed well- known statistical model may not give a very satisfactory fit for a collected data set. This might be due to typical inherent behavior of the phenomenon that gives rise to the specific data in hand and the model attempted being incapable of capturing such a behavior. The epsilon-skew normal density proposed by Mudholkar and Hutson (2000) gives better fit for certain data sets and can be obtained by the CT of a Normal density.

In the literature some methods to modify a model to give a better fit for the data set have been reported. Marshal and Olkin (1997) have extended the class of exponential and Weibull family by introducing an additional parameter. Fernandez and Steel (1998) have proposed a technique of generating unimodal skewed distributions from a symmetric unimodal distributions by using a scalar parameter.

New families of distributions have also been introduced using the concepts of conditioning or truncation. For example one may refer to Azzalini (1985) and Azzalini and Dalla Valle (1996) and Arnold et al. (2002).

Fernandez, *et al.* (1995) have proposed υ- spherical multivariate densities and studied their robustness properties. Rattihalli and Basugade (2008) have generated a class of multivariate densities by using contour transformation. Basugade (2015) have generated densities based on the size of Contours of a continuous density function f.

Here we generate densities based on the size of Contours of discrete probability distributions. As an Illustration contour densities of Geometric distribution have been discussed.

Basic concepts:
Let f(x), $x \in \Re^k$ be a p. d. f. and zero be the modal value. For $0 \le u \le f(0)$, the set {x: f (x) = u} is called as a contour of f, or an u-level set of f. However for convenience the set

$C_f(u) = \{x: f(x) \ge u\}$ (1)

be called the u-contour of f and

$C_f(0)$=support of f = $\lim_{k\to\infty}\{c_f(u_k)\}$, (2)

where u_k is any sequence decreasing to zero.

Contour Transformation: Let f be a density function with modal value 0. For $0 \le u \le f(0)$ we consider a transformation of contour $C_f(u)$ to $C^*(u)$ such that

i) the class $\{C^*(u) : 0 \le u \le f(0)\}$ is non-increasing in u (3)

and

ii) $\wedge(C_f(u)) = \wedge(C^*(u))$, $0 \le u \le f(0)$. (4)

where, $\wedge$ is the Lebesque measure on $\Re^k$. Note that corresponding to such a class $\mathbf{C}^*$ = $\{C^*(u): 0 \le u \le f(0)\}$ there exists a function f^* (say) given by $f^*(x) = \sup\{u: x \in C^*(u)\}$ such that $C^*(u) = C_{f^*}(u)$, $0 \le u \le f(0)$. It is to be noted that, from condition (3), corresponding to the class $\mathbf{C}^*$, there exists a function f^* and is p. d. f.. The requirement (3) is a major constraint on a CT. An arbitrary transformed class of contours may not correspond to a function.

Contour Transformation:

A Contour Transformation transforms each member $C_f(u)$ of the class $\mathbf{C}_f$ to $C^*(u)$, so that $\wedge(C_f(u)) = \wedge(C^*(u))$ for $0 \le u \le f(0)$ and empty set is transformed to empty set itself, so that $C^*(u) = C_{f^*}(u)$, for some density f^*. The p.d.f. f^* is said to be obtained by a CT of the p.d.f. f and is denoted by CT(f).

Densities using discrete probability distributions:

The concept of contours and contour transformation can be extended to discrete models. For notational convenience we shall consider discrete distributions with support {0,1,2, - - -}. Let (p_0, p_1, p_2, - - -) be a discrete probability distribution. We define a function p(x)

$$p(x) = \sum_{i=0}^{\infty} p_i \, [i \le X < i+1]$$

where $[\,A\,]$ is the indicator function of the set A.

Note that p(x) is a p. d. f. and corresponding distribution function is

$$F(x) = \sum_{i \le x-1} p_i + (x - [x])\, p_{[x]}$$

where [x] is integer part of x.

The u-contour of p is $C_p(u) = \{i: p_i \ge u\}$ and define $h(u) = \mathbf{C}(C_p(u))$,

where $\mathbf{C}$ is the counting measure that counts the number of elements in the set. Hence

$$h(u) = \mathbf{C}(C_p(u))$$

$$= \sum_{i=0}^{\infty} [p_i \ge u], \quad 0 < u \le \max\{p_i\} \qquad (5)$$

Note that for any unimodal distribution (p_0, p_1, p_2, - - -, p_m, p_{m+1}, - - -) with modal value m we have

$$h(u) = \begin{cases} k_2 - k_1 & o \le u \le p_m \\ 0 & u > p_m \end{cases} \qquad (6)$$

where, k_1 and k_2 are such that

$p_{k_1-1} < u < p_{k_1}$ and $p_{k_2} < u < p_{k_2-1}$. That is k_1 and k_2 are such that $p(k_1) \ge u$ and $p(k_2 - 1) \ge u$.

Illustration:

Geometric distribution:

$p_k = q^k p$

Then the h(u) function is

$$h(u) = \begin{cases} \lfloor \log_q(u/p) \rfloor + 1, & 0 < u \le \max\{p_k\} \\ 0, & u > p_0 = p \end{cases} \qquad (7)$$

Where [log(u/p)] indicate the integer part of log (u/p).

Conclusions:

By using concept of Contour transformation we can generate densities based on the size of contours even for a discrete probability distribution.

For example, if p_0 = 0.2, p_1 = 0.3, p_2 = 0.1 and p_3 = 0.4 then the graph of p(x) and h(u) are as given in fig. 1 and 2.

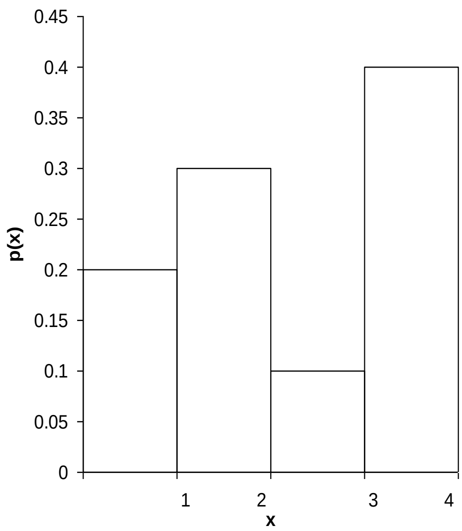

Fig. 1: Graph of p(x)

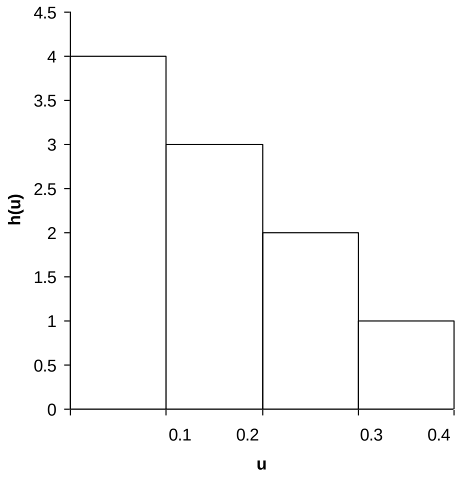

Fig. 2: Graph of h(u)

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