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MARKOVIAN QUEUE WITH SERVICE INTERRUPTION UNDER N-POLICY

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Abstract:
This investigation deals with a markovianqueue with service interruption under N-policy. According to the N-policy, the server is turned on only when there are 'N' or more customers are encountered in the system. The broken-down and repair times of the server are assumed to be exponentially distributed. Various performance measures are determined using recursive method. Numerical results are provided. At last, conclusions are given.
Keywords: N-policy, Markovian Model, Service Interruption, Queue Size.

1. Introduction

In any production system in which the service facility starts processing after accumulation of a fixed number of jobs, is known as N-policy system. The production facility may be subject to breakdowns any time when the production facility is in operation or not. The concept of the N-policy was first introduced by Yadin and Naor (1963). Ke and Wang (2002) focused on a single removable server queueing system with finite capacity operating under N policy. They have provided a recursive approach for developing the steady-state probability distributions of the number of customers in the system. Threshold N-Policy for queueing system with un-reliable server and vacationswas studied by Sharma (2010). Yang et al. (2013)analyzed a repairable M/M/1/N queueing system under a threshold-based recovery policy.

The modeling for unreliable system has been done by many queueing theorists and a considerable amount of works in different frameworks has appeared in literature from time to time. Queueing models with service interruption have successfully used in abundant congestion problems ranging from day-to-day to many industrial scenarios too. An M/G/1 queue with second optional service and server breakdowns was studied by Wang (2004). Notable contributions can be seen in the work of Jain *et al.* (2011, 2012a,b, 2013) and Sharma (2015).

In this paper, we develop amarkovian queuewith service interruption under N-policy. The rest of this paper is organized as follows. In section 2, we give a description of the queueing model along with governing equations of the model. The recursive method is provided in section 3. In section 4, we provide the various performance measures. In section 5, numerical results are presented. Finally, conclusions are given in Section 6.

2. Model Description

Consider a markovianM/M/1 queue with service interruption under N-policy. The server is turned on when 'N' or more customers are accumulated in the system and it turns off when the queue becomes empty.The customers arrive at service station singly according to the Poisson fashion with arrival rate λ. The server provides the service to all arriving customers with rate μ_b on the first come first serve (FCFS) basis. During busy state, the service may be interrupted with rate α. As the breakdown occurs, it is immediately sent for repairing at repair facility with repair rate β.

Notations:

λ	:	Arrival rate
μ_b	:	Service rate during busy period
α	:	Failure rate
β	:	Repair rate
N	:	Threshold Parameter
K	:	Finite capacity of the system

$P_{i,n}(0 \le n \le K)$:Steady state probability that there are 'n' customers in the system when the server is in ith (i=0,1,2) state.

Now, we provide the governing equations of the queueing model using the Markov theory as follows:

$$\lambda P_{0,0} = \mu_b P_{0,1} \quad (1)$$

$$\lambda P_{0,n} = \lambda P_{0,n-1}, \qquad 1 \le n \le N-1 \quad (2)$$

$$(\lambda + \mu_b + \alpha) P_{1,n} = \mu_b P_{1,n+1} + \lambda P_{1,n-1} + \beta P_{2,n}, \quad 1 \le n \le N-1 \quad (3)$$

$$(\lambda + \mu_b + \alpha) P_{1,N} = \mu_b P_{1,N+1} + \lambda P_{1,N-1} + \lambda P_{0,N-1} + \beta P_{2,N} \quad (4)$$

$$(\lambda + \mu_b + \alpha)P_{1,n} = \mu_b P_{1,n+1} + \lambda P_{1,n-1} + \beta P_{2,n}, \qquad N+1 \le n \le K-1 \qquad (5)$$
$$(\mu_b + \alpha)P_{1,K} = \lambda b_{K-1} P_{1,K-1} + \beta P_{2,K} \qquad (6)$$
$$(\lambda + \beta)P_{2,n} = \lambda b_{n-1} P_{2,n-1} + \alpha P_{1,n}, \qquad 1 \le n \le K-1 \qquad (7)$$
$$\beta P_{2,K} = \lambda P_{2,K-1} + \alpha P_{1,K} \qquad (8)$$

3. Recursive Method

In this section, we apply recursive method for obtaining the steady state probabilities in the following theorem.

Theorem 1: The steady-state probabilities for Idle, busy and repair states respectively are given as:

$$P_{0,n} = \frac{\Theta_n}{\sum_{i=0}^{N-1}\Theta_i + \sum_{i=1}^{K}\xi_i + \sum_{i=1}^{K}\zeta_i}, \qquad 0 \le n \le N-1 \qquad (9)$$
$$P_{1,n} = \frac{\xi_n}{\sum_{i=0}^{N-1}\Theta_i + \sum_{i=1}^{K}\xi_i + \sum_{i=1}^{K}\zeta_i}, \qquad 1 \le n \le K \qquad (10)$$
$$P_{2,n} = \frac{\zeta_n}{\sum_{i=0}^{N-1}\Theta_i + \sum_{i=1}^{K}\xi_i + \sum_{i=1}^{K}\zeta_i}, \qquad 1 \le n \le K \qquad (11)$$

where

$$\Theta_n = \Theta_{n+1}, \quad n = N-1, N-2, N-3, \dots, 0$$
$$\xi_0 = 0, \quad \xi_1 = \frac{\lambda\Theta_0}{\mu_b},$$
$$\xi_n = \frac{(\lambda + \mu_b + \alpha)\xi_{n-1} - \lambda\xi_{n-2} - \beta\zeta_{n-1}}{\mu_b}, \quad n = 2,3,\dots,N,$$
$$\xi_n = \frac{(\lambda + \mu_b + \alpha)\xi_{n-1} - \lambda\xi_{n-2} - \beta\zeta_{n-1}}{\mu_b}, \quad n = N+1, N+2, \dots, K,$$
$$\zeta_0 = 0, \ \zeta_n = \frac{(\lambda + \beta)\zeta_{n+1} - \alpha\xi_{n+1}}{\lambda}, \qquad n = 1,2,\dots,K.$$

Proof: Using the normalization condition$\sum_{i=0}^{N-1}\Theta_i + \sum_{i=1}^{K}\xi_i + \sum_{i=1}^{K}\zeta_i = 1$ and solving equations (1)-(8) recursively, we obtain the results of theorem 1.

4. Performance Measures

In this section, we establish expressions for performance measures which are given as:

- The probability of the server beingIdle is

$$PI = \sum_{n=0}^{N-1} P_{0,n} \qquad (12)$$

- The probability of the server on busy state is

$$PB = \sum_{n=1}^{K} P_{1,n} \qquad (13)$$

- The probability of the server under repair state is

$$PD = \sum_{n=1}^{K} P_{2,n} \qquad (14)$$

- The average queue length is

$$LQ = \sum_{n=1}^{N-1}(n-1)P_{0,n} + \sum_{n=1}^{K}(n-1)(P_{1,n} + P_{2,n}) \qquad (15)$$

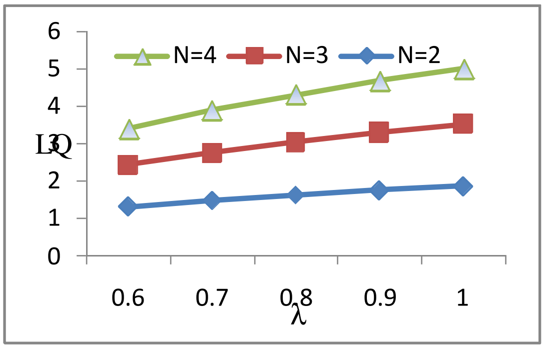
5. Numerical Results

In this section,we provide numerical results for validation purpose. The default parameters for table andfigs are chosen as λ=0.9, μ_b=2, α=0.1, β=2, N=4, K=7.

Table 1

α	K=3			K=5		
	PI	**PB**	**PD**	**PI**	**PB**	**PD**
0.1	0.807	0.021	0.170	0.813	0.069	0.116
0.2	0.804	0.022	0.173	0.810	0.070	0.120
0.3	0.802	0.023	0.175	0.806	0.071	0.124
β	**PI**	**PB**	**PD**	**PI**	**PB**	**PD**
1.0	0.804	0.023	0.173	0.810	0.072	0.120
2.0	0.807	0.022	0.170	0.813	0.071	0.116
3.0	0.808	0.021	0.170	0.815	0.070	0.115

Table 1 shows that PB and PD increase (decrease) with the increasing values of α(β). Further, PI shows the reverse trend for increasing values of α(β). Figs 1-2 depict the behavior of LQ for different values of N by varying λ and μ_b. Fig 1dipictsthat the average queue length increases on increasing the values of arrival rate, which is quite obvious. From fig 2, we note that the trend of LQ is decreasing slightly with the increasing values of service rate. Further, the increasing trend is observed for LQ with the increasing values of N.



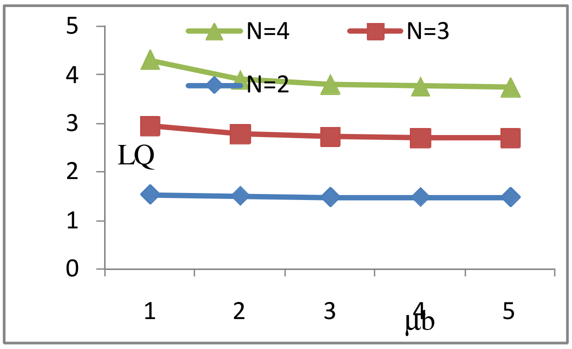


Fig. 1: Effect of λon LQ for different values of N.**Fig. 2:** Effect of $\mu_b$on LQ for different values of N.

Overall, we conclude that the average queue length increases remarkably as arrival rate and breakdown rate increase.

5. Conclusions

In this paper, we have analyzed markovian queue with service interruption under N-policy. Our results can be treated as performance evaluation tool for the concerned system which may be suited to many congestion situations arising in many practical applications encountered in computer and communication systems, distribution and service sectors, production and manufacturing systems, etc.

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